Escape from a poverty trap in an AK growth model

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Abstract

If total factor productivity falls below a critical minimum threshold, the AK growth model exhibits a poverty trap. A fully-funded investment subsidy lifts the economy out of the trap to a saddle-stable steady state - an unfamiliar outcome in the AK framework.

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1. Introduction

In a recent paper Azariadis and Stachurski (2005) have reviewed the literature on poverty traps, affirmed the possibility of their existence, and reached the "optimistic" conclusion that "poverty is not the result of some simple geographic or cultural determinism…. Temporary policy shocks will have large and permanent effects if one-off interventions can cause the formation of new and better equilibria" (2005 p.374). They acknowledge, however, that "traps which prevent growth and prosperity cannot be overcome without proper understanding and the careful design of policy" (2005 p.374).

This paper sets up a very simple poverty-trap model for an economy below a critical technology threshold, and introduces an equally simple policy intervention: a targeted investment subsidy funded by a general output tax. We find that the policy both breaks the poverty trap and establishes a saddle-stable steady state for the economy. The sustainability of this steady state is contingent on maintaining the subsidy regime in perpetuity, in contrast to the Azariadis-Stachurski prospect of permanent effects from a temporary policy shock. Our result suggests that their self-sustaining escape-from-poverty outcome requires either increasing returns to scale or a permanent exogenous change in total factor productivity.

To our knowledge, this is a new approach to the analysis of policy interventions in the *AK* model. King and Rebelo (1990) have explored the effect on the growth rate of taxes on labour and capital, but their analysis was limited to the predictable negative effects of such taxes and did not incorporate a government budget constraint, so that their tax revenues go to waste. Ligthart and Ploeg (1994) and Rubio and Aznar (2002) have considered the consequences of interacting the endogenous growth framework with pollution by building on a modeling device first provided by Barro (1990), which investigates how public funding can interact with the *AK* technology.

The innovation in Barro's work lay in a reduced-form assumption which related the linear productivity term, *A*, positively to the level of public spending, with an *ad hoc* assumption of diminishing marginal benefit. The drawback is that because it is a reduced-form, Barro's model is silent on how the endogenous growth framework would react when presented with a specific investment policy prescription.

In contrast, without resorting to any functional assumption on the relationship between the technology and the public policy under scrutiny, we offer a model in which the investment policy regime, the incentives faced by the household, and the fiscal constraint placed on the government, are simple and explicit. We derive the intertemporal equilibrium outcome for that environment.

2. The Model

2.1 The AK Poverty Trap

Consider an infinite-horizon economy that consists of a continuum of households with unit mass. Each household wishes to maximize its discounted life-time utility

$$\underset{c}{Max.} \int_0^\infty \ln(c) e^{-\rho t} dt \,,$$

where c is consumption and ρ is the pure time rate of preference[1]. The representative household faces an instantaneous resource constraint which states that the total amount of output is either consumed or invested:

$$c + i = Ak \,, \tag{1}$$

where i is investment, k is capital, and A is the capital-output ratio. Capital accumulates according to

$$\dot{k} = i - \delta k \,, \tag{2}$$

with a given initial capital stock k_0 and depreciation rate δ.

The equilibrium for this economy can be characterized by the following system of differential equations:

$$\dot{c} = c[A - \delta - \rho],$$

and

$$\dot{k} = Ak - c - \delta k \,.$$

The standard assumption in the growth literature is that

$$A > \delta + \rho \,,$$

that is, the production technology is sufficiently efficient to yield an equilibrium trajectory of this economy which exhibits sustained growth at the positive, constant rate $A - \delta - \rho > 0$.

There is, in fact, no reason in principle to believe that a real-world economy necessarily satisfies the required technological assumption for sustainable growth. The

[1] All italicised roman symbols represent real, per capita magnitudes. Greek symbols represent key parameters.

converse of the above result is that if an economy does not have a sufficiently advanced technology

$$A < \delta + \rho , \tag{A1}$$

then the economy experiences sustained decay and ultimately falls towards a poverty trap with zero capital stock.

2.2 Investment Subsidy

In this section we evaluate a targeted investment-subsidy regime, financed by an output tax, under the premise of (A1). We demonstrate that there exists a well-defined, saddle-node, steady state, which provides the otherwise "doomed" economy with a deterministic trajectory away from the potential equilibrium trap. The inferior technology assumption (A1) is retained throughout the analysis.

Let σ represent the marginal rate of investment subsidy funded by the government, and q denote the lump-sum amount of tax collected to finance the subsidy. Then the household's budget constraint becomes

$$c + i = Ak - q \tag{1'}$$

and the capital transition equation becomes

$$\dot{k} = (1+\sigma)i - \delta k . \tag{2'}$$

From the household point of view, σ is exogenous, as is the initial amount of capital k_0.

Applying the Maximum Principle to the problem yields the following set of differential equations that describe the household's optimizing behavior:

$$\dot{c} = c[(1+\sigma)A - \delta - \rho], \tag{3}$$

and

$$\dot{k} = (1+\sigma)(Ak - q - c) - \delta k \ . \qquad (4)$$

We next turn our attention to the government budget constraint and define the equilibrium for this economy.

2.3 Government Budget Constraint

The government uses the output tax to finance the investment subsidy scheme at every instant. This fiscal operation can be succinctly expressed by the following government budget constraint

$$\sigma i = q \ . \qquad (5)$$

With the total amount of output tax q collected being exogenous, conditions (1') and (5) determine the equilibrium marginal subsidy rate as

$$\sigma = \frac{q}{i} = \frac{q}{Ak - q - c} \ . \qquad (6)$$

Definition: *The economy's equilibrium can be defined as the capital and consumption trajectories that satisfy conditions (3), (4), and (6). The boundary conditions for the dynamic system are* $k(0) = k_0$ *and* $\lim_{t \to \infty} k(t) = k_s$*, where* k_s *is the steady-state capital stock.*

2.4 Equilibrium Analysis and Steady State

In this section we first solve for the steady state, and discuss the properties of the transition dynamics en route towards the long-run equilibrium.

Substitute the government budget constraint (6) into the household's first-order conditions (3) and (4) to condense the dynamic system:

$$\dot{c} = c\left\{\left[1+\frac{q}{Ak-q-c}\right]A-\delta-\rho\right\} \tag{7}$$

and

$$\dot{k} = \left[1+\frac{q}{Ak-q-c}\right](Ak-q-c)-\delta k = Ak-c-\delta k\ . \tag{8}$$

Setting (7) and (8) simultaneously to zero shows that the economy now has a steady state[2]. For the consumption differential equation, we have

$$\dot{c} = 0 \Rightarrow c = \frac{q(\delta+\rho)}{A-(\delta+\rho)}+Ak\ , \tag{9}$$

assuming $c \neq 0$. Given (A1), the first term in (9) is negative

$$\frac{q(\delta+\rho)}{A-(\delta+\rho)} < 0\ . \tag{10}$$

For the capital differential equation, we have

$$\dot{k} = 0 \Rightarrow c = Ak-\delta k\ . \tag{11}$$

Solving conditions (9) and (11) simultaneously gives the unique steady-state level of capital

$$k_s = -\frac{1}{\delta}\frac{q(\delta+\rho)}{A-(\delta+\rho)} > 0\ ,$$

[2] The usual AK model, presented in (1), does not yield any steady state.

given (10).

Provided that $A > \delta$ (that is, the economy's output is more than sufficient to merely replace depreciated capital) then (11) identifies a corresponding unique, steady-state level of consumption $c_s = (A - \delta)k_s$.

2.5 Stability Analysis

Having derived the unique steady state, we now investigate the dynamic properties when the economy is away from its steady state.

Proposition. *The unique steady-state* (k_s, c_s) *is locally, saddle-path stable.*

Proof. To demonstrate saddle-path stability, we first obtain the elements of the appropriate Jacobian matrix J evaluated at the steady state: $\frac{\partial \dot{k}}{\partial k} = A - \delta$, $\frac{\partial \dot{k}}{\partial c} = -1$, $\frac{\partial \dot{c}}{\partial k} = -\frac{c_s q A^2}{(Ak_s - q - c_s)^2}$, and $\frac{\partial \dot{c}}{\partial c} = \frac{c_s q A}{(Ak_s - q - c_s)^2}$. It is straightforward to show that the determinant of the Jacobian is negative: $\det J = -\frac{c_s q A \delta}{(Ak_s - q - c_s)^2} < 0$. This means that the relevant eigenvalues must have opposite signs, which establishes saddle-path stability within the neighborhood of the steady state.

Q.E.D.

2.6 Phase Diagram

We now construct the phase portrait for our economy to reinforce our stability conclusion in the previous section and extend our analysis to global dynamics. Figure 1 shows the stationary isocline for consumption from condition (9), with slope *A* and with the negative intercept required by (10). According to (7), any point above (below) the stationary locus experiences upward (downward) pressure, as illustrated by the broken arrows.

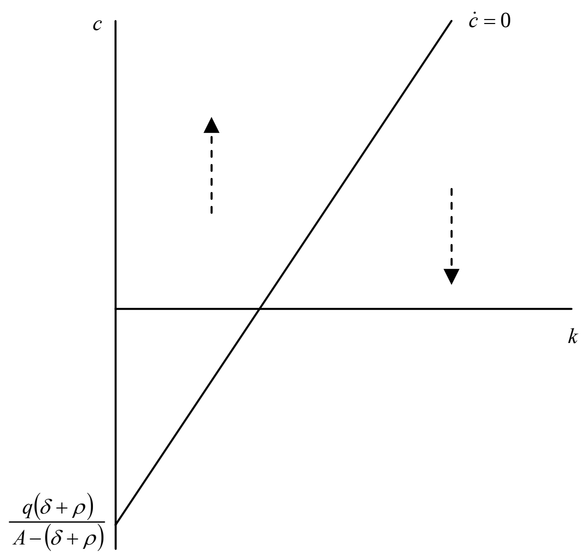

Fig. 1: Stationary Consumption Locus

Figure 2 shows the stationary capital locus from (11), passing through the origin and with slope $A-\delta$. According to (8), any point above (below) the stationary locus experiences leftward (rightward) pressure, as illustrated again by the broken arrows.

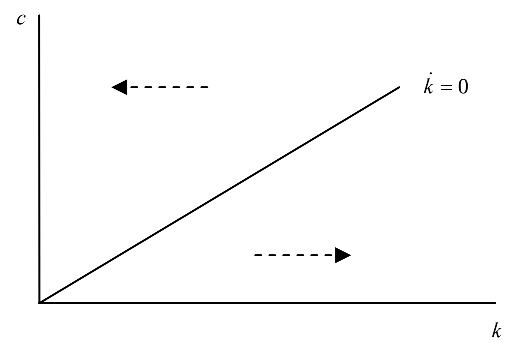

Fig. 2: Capital Stationary Locus

Combining the two loci, the final phase-portrait is drawn as Figure 3:

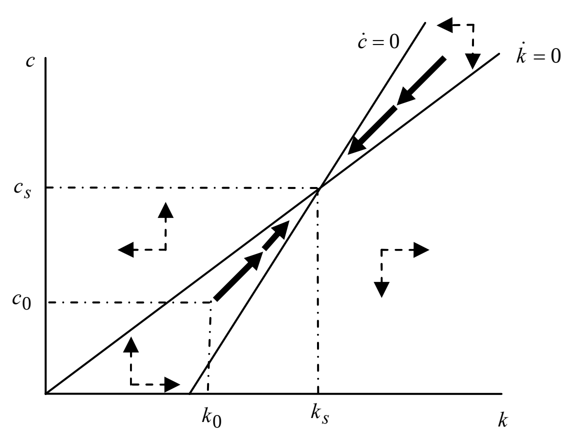

Figure 3: Final Phase Portrait

This illustrates the global saddle-path (dark arrows) of the dynamic system, which echoes our local, analytical finding in the previous section. Given an initial capital stock k_0, and imposing the transversality condition that the economy approaches the steady state in the long run, the saddle path allows us to uniquely determine the corresponding, endogenous consumption level at the initial date c_0.

3. Welfare Considerations

Depending on the presence or absence of the proposed policy, two alternative steady states are possible. Welfare analysis requires direct comparison of $\int_0^\infty \ln(c^*) e^{-\rho t} dt$ with $\int_0^\infty \ln(c^{**}) e^{-\rho t} dt$, where the single asterisk denotes no-policy and two asterisks denote with-policy consumption equilibrium trajectories. The nonlinearity of (7), *inter alia*, makes the above direct comparison an analytically formidable, if not impossible, task. Of course, modelers often substitute numerical simulations, which are inherently sensitive to calibration, when such a problem arises. Given the theoretically-oriented framework we have constructed up to this point, we do not consider that such a numerical exercise would usefully expand the ambit of our analysis.

Nevertheless, we can still gain a considerable degree of insight into welfare consequences by observing that in the absence of the investment-subsidy regime and under the assumption (A1), the economy's consumption trajectory is governed by

$$\dot{c} = c(A - \delta - \rho) \Rightarrow \frac{\dot{c}}{c} = A - \delta - \rho < 0,$$

which entails that the economy asymptotically approaches zero consumption as time elapses. With logarithmic utility, this trajectory implies extreme dissatisfaction for households in the future, since as $c(t) \to 0$, $\ln(c(t)) \to -\infty$.

In contrast, the equilibrium conditions described in sections 2.4 and 2.5, with the subsidy regime, entail $\lim_{t \to \infty} c(t) = c_s > 0$, which yields a permanent stream of positive utility in the equilibrium.

The steady-state welfare with the policy almost certainly dominates the steady-state welfare without the policy, all else equal. Even discounted back to the present, the stream of utility for the without-policy scenario will be negative infinity while that for the with-policy scenario will be positive. Notwithstanding the analytical difficulty, the proposition that the household should prefer a transition path leading to a sustainable future to one that leads to ultimate extinction has intuitive appeal.

In effect the tax-and-subsidy package is a compulsory saving scheme which results in a permanently higher level of investment, but initially lower level of consumption, than households optimally choose in the without-subsidy situation. The individual household cannot unilaterally introduce the tax/subsidy package, which therefore is not included in the feasible set of options when it solves its maximising problem in the simple *AK* setting. We conjecture that if the coordination issue were resolved and the utility-maximising household were offered the chance to vote for the policy to be implemented, it would do so.

4. Conclusion

The analysis in this paper demonstrates that in an *AK* economy locked into a poverty trap due to low total factor productivity, introduction of a simple investment subsidy simultaneously brings into existence a steady state with positive capital stock and consumption, and enables the economy's households to move onto a saddle path leading to that steady state.

The subsidy regime must be permanent, not temporary, if the steady state is to be sustained. However, if there is some path-dependence, the poor economy's policy-driven escape from its poverty trap may subsequently be locked-in by changes in expectations, institutions, or attitudes; or by a shift in the economy's technical efficiency bringing about an increase in A sufficient to reverse the sign in (A1).

It has not been possible to offer a formal analytical proof that the policy intervention is welfare-enhancing, notwithstanding its intuitive attractiveness.

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